

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/267643310>

# Aided Inertial Measurement System Applied to Torpedo Pile Trajectory Reconstruction

Conference Paper · January 2011

DOI: 10.1115/OMAE2011-49080

CITATIONS

0

READS

31

4 authors, including:



Rodrigo Sauri Lavieri

University of São Paulo

9 PUBLICATIONS 2 CITATIONS

SEE PROFILE



Eduardo A. Tannuri

University of São Paulo

124 PUBLICATIONS 440 CITATIONS

SEE PROFILE



André L. C. Fuarra

Federal University of Santa Catarina, Joinville, ...

103 PUBLICATIONS 554 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



Floating ocean systems hydrodynamics [View project](#)



Dynamics of ocean vessels [View project](#)

All content following this page was uploaded by [Eduardo A. Tannuri](#) on 18 February 2015.

The user has requested enhancement of the downloaded file.

OMAE2011-49080

## AIDED INERTIAL MEASUREMENT SYSTEM APPLIED TO TORPEDO PILE TRAJECTORY RECONSTRUCTION

**Rodrigo Sauri Lavieri**  
Numerical Offshore Tank (TPN)  
University of São Paulo  
São Paulo, SP, Brazil

**Eduardo Aoun Tannuri**  
Numerical Offshore Tank (TPN)  
University of São Paulo  
São Paulo, SP, Brazil

**André L.C. Fuarra**  
Numerical Offshore Tank (TPN)  
University of São Paulo  
São Paulo, SP, Brazil

**Diego Cascelli Corrêa**  
PETROBRAS E&P  
Naval Engineering  
Rio de Janeiro, RJ, Brazil

### ABSTRACT

Torpedo pile has become one of the most popular foundation systems in Brazil's offshore oil exploitation, mostly, due to its low installation cost. Additionally, torpedo piles have shown good fixation capability even when applied in mooring configuration such as taut leg with relatively close angles to the vertical. This fixation capability is closely dependent to the depth of penetration and the final attitude of the pile. In order to determine these parameters, MEMS-based Inertial Measurement Units have been used. Units of this kind are known for high noise density and, alone, are not adequate to any kind of navigation.

Concerning these limitations, other sources of data are added to the inertial measurements, in order to improve the system state estimation. This data fusion is carried out by the Kalman Filtering and is also called Aided Inertial Measurement Units.

This paper presents the Kalman Filter implementation and the results obtained from the fusion of the pressure gage signal, alternative pitch and roll measurements and the inertial measurements applied to the torpedo pile deployment.

### INTRODUCTION

Inertial navigation systems have been used since the beginnings of submarine vehicles, at the first half of the 20<sup>th</sup> century. Even today, ROVs (Remotely Operated Underwater Vehicle) applied on the offshore industry counts on their accelerometers and gyros to help the navigation process. However, the sensors used in that kind of application are expensive and, in some countries, unavailable.

MEMS (Micro-Electro-Mechanical Systems) technology provides rough and cheap sensors allowing the development of low-cost inertial measurement units (IMU). This cost reduction encourages the use of this kind of device in a wider range of applications, e.g. to equip anchors.

The present research started investigating an IMU that was applied to monitor the launching of the torpedo-pile. It is a MEMS-based unit, firstly developed to obtain the torpedo aped and final tilt. Figure 1 displays the IMU attached to the pile's body.

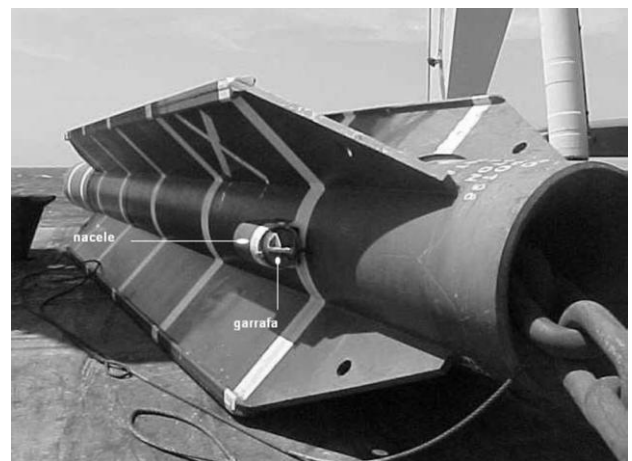


Figure 1 - IMU in the nacelle attached to the torpedo pile

This IMU is equipped with 3 accelerometers, 3 rate gyros, temperature sensor and a pressure gage.

During the first steps of the research, the performance of each sensor was studied as well as the algorithms to process the data acquired; the results are presented in [1]. During this phase, the limitations of the apparatus concerning the trajectory reconstruction was observed.

Aided inertial navigation proved to be the solution. This technique consists in joining other sources of information to improve the parameters estimate. Also called sensor fusion, the method applies an algorithm based on Kalman Filtering.

As no extra source of data have been added to the actual instrumentation, no great results enhancement is expected. However, due to the better performance of the accelerometers over the rate gyros, some gain, especially on tilt estimate, is expected. Moreover, once the algorithm is implemented, adding new sources of measurements is a trivial procedure.

## NOMENCLATURE

$\alpha_{X,Y,Z}$	Accelerometer X,Y,Z signal (acceleration in body frame)
$\theta$	Inclination Angle (from the vertical)
$\psi$	Spin angle (Euler's description)
$\phi$	Precession angle (Euler's description)
$g$	Gravity acceleration (scalar)
$x$	States of the system

## EXTENDED KALMAN FILTER

The Kalman Filter is a state estimator that, under some hypothesis, gives an optimal solution. It is based on two models, one for the system dynamics and the other for the measurements. Besides the regular dynamic model, it also has non deterministic terms that will be propagated during the integration.

Commonly, the filter is divided into two separated steps, propagation and update.

During the propagation step, the whole set of equations are integrated. Without any other source of data, the integration results are the best estimate for the set of state and parameters of the system (called *a priori* estimate). As soon as a measurement is taken, the filter evaluates the reliability of the information, through its covariance matrix ( $R$ ), and corrects the *a priori* estimate. This step is called update and the new estimated values are known as *a posteriori* estimates. In this work, the output from the propagation step (*a priori*) will have the superscript “-” and after an update step (*a posteriori*), the estimate will have the superscript “+”.

The Extended Kalman Filter (EKF) is an adaptation of the regular filter, useful to deal with non linear problems such as the strapdown inertial navigation. The main characteristic of the EKF version is that at every step of the integration (propagation), the system and measurement equations are linearized near the last (and best) estimate.

## FILTER STRUCTURE

As said before, strapdown INS (Inertial Measurement System) computers perform three sets of calculus, those for computing direction changes of the sensors sensitive axes, the gravity acceleration correction and the time integration. This partition indicates that it is also possible to share the filtering process into blocks. Here, two distinct filters were implemented. The first one deals with the attitude estimation while the second one computes the translational motion and the gravity correction. As will be seen further, this strategy leads to smaller system matrixes that are easier to invert. Figure 2 illustrates the division suggested before.

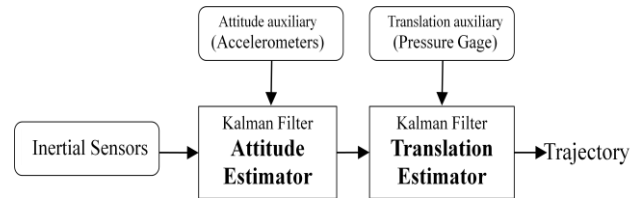


Figure 2 – Filter Structure diagram

It is worth noticing that there will be an open loop process or, in other words, outputs from the translational block will not be fed-back to update system matrixes.

Being  $P$ , the system error's covariance matrix, equations (1) to (8) summarize the filtering steps.

$$x_1^+ = x_0 \quad (1)$$

$$P_1^+ = Q \quad (2)$$

$$x_{t+dt}^- = x_t^+ + \int_t^{t+dt} f dt \quad (3)$$

$$P_{t+dt}^- = P_t^+ + \int_t^{t+dt} \dot{P} dt \quad (4)$$

Where:

$$\dot{P} = FP + PF^T + Q \quad (5)$$

During the update cycle, the equations are the following:

$$K_t = P_t^- H^T (H P_t^- H^T + R)^{-1} \quad (6)$$

$$x_t^+ = x_t^- + K_t (z - h(x, t)) \quad (7)$$

$$P_t^+ = P_t^- - K_t H P_t^- \quad (8)$$

The algorithm manages to select which information to use by the variable  $K$ , called Kalman's Gain.

One may notice that the state propagation applies the set of equations  $f$ , that are fully nonlinear. In turn, the covariance matrixes are propagated using a linearized form of the

equations identified as  $F$ . Similarly,  $h$  is the array of nonlinear measurement equations while  $H$  is its linear matrix form. This strategy, in addition to the high order integration method, revealed improved stability.

All the variables will be detailed henceforth.

## SYSTEM MODELS

The states chosen to describe the system are presented below:

$$x_{attitude} = \begin{bmatrix} a \\ b \\ c \\ d \\ bias_{gx} \\ bias_{gy} \\ bias_{gz} \end{bmatrix} \quad (9)$$

$$x_{translation} = \begin{bmatrix} x \\ y \\ z \\ u \\ v \\ w \\ bias_{gx} \\ bias_{gy} \\ bias_{gz} \end{bmatrix} \quad (10)$$

Being  $a, b, c, d$  the components of the quaternion,  $x, y, z$  the Cartesian position and  $u, v, w$  the velocity components in the body reference system.

This version of the filter will not estimate the sensors scale factor. This important parameter will be considered constant and will be used to correct signal before it enters the filtering routine.

Some system parameters were also included in the state vector. States (9) and (10) follow the separation suggested before.

The system dynamics is given by equation (11).

$$\dot{x} = f(x, t) + W \quad (11)$$

Considering the attitude block, the array of equations (12) describes the system dynamics.

$$f_a = \begin{bmatrix} 0,5 \\ \begin{bmatrix} -b & -c & -d \\ a & -d & c \\ d & a & -b \\ -c & b & a \end{bmatrix} \begin{bmatrix} p - bias_{gx} \\ q - bias_{gy} \\ r - bias_{gz} \end{bmatrix} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

Where the propagation of quaternions is given by equation (13) [2] and the three rate gyro bias are assumed constant.

$$\dot{q} = 0,5 \begin{bmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & -c & b & a \end{bmatrix} \begin{bmatrix} 0 \\ p \\ q \\ r \end{bmatrix} \quad (13)$$

In order to perform the propagation of the system error covariance matrix ( $P$ ), the linearized form of  $f$  is demanded. Being  $x_k$  the last estimate of the states,  $F$  is given by equation (14).

$$F = \frac{\partial f}{\partial x} \Big|_{x_k} \quad (14)$$

Executing the differentiations leads to expression (15).

$$F_a = 0,5 \begin{bmatrix} 0 & (bias_{gx} - p) & (bias_{gy} - q) & (bias_{gz} - r) & b & c & d \\ -(bias_{gx} - p) & 0 & -(bias_{gz} - r) & (bias_{gy} - q) & -a & d & c \\ -(bias_{gy} - q) & (bias_{gz} - r) & 0 & -(bias_{gx} - p) & -d & -a & b \\ -(bias_{gz} - r) & -(bias_{gy} - q) & (bias_{gx} - p) & 0 & c & -b & -a \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (15)$$

The dynamic equations for the translational states are given by (16).

$$f_t = \begin{bmatrix} (a^2 + b^2 - c^2 - d^2)u + 2(bc - ad)v + 2(bd + ac)w \\ 2(bc - ad)u + (a^2 - b^2 + c^2 - d^2)v + 2(cd - ab)w \\ 2(bd + ac)u + 2(cd - ab)v + (a^2 - b^2 - c^2 + d^2)w \\ \alpha_x - 2(bd + ac)g - bias_{ax} \\ \alpha_y - 2(cd - ab)g - bias_{ay} \\ \alpha_z - (a^2 - b^2 - c^2 + d^2)g - bias_{az} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

After linearization and arranged in matrix form, expression (17) is produced.

$$F_t = \begin{bmatrix} 0 & 0 & 0 & (a^2 + b^2 - c^2 - d^2) & 2(bc - ad) & 2(bd + ac) & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(bc - ad) & (a^2 - b^2 + c^2 - d^2) & 2(cd - ab) & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(bd + ac) & 2(cd - ab) & (a^2 - b^2 - c^2 + d^2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (17)$$

The subscripts “a” and “t” here indicate if the model is related to the attitude block or the translation block.

## MEASUREMENT MODEL

The measure model is the ensemble of equations that relates the measures available with the states of the system. Although these equations do not represent the dynamic of the system, they can be nonlinear as will be seen further on. In this case, as with the system equations, they must be linearized near the last estimate.

The connection between sensor fusion and the Kalman Filtering is due to its capability of assembling many measurement sources through the measurement model.

In the present research, no new sensor was added to the main IMU. Nevertheless, more information can be drawn from the same set of sensors. They are: pitch and roll estimates using gravity components reading; the coordinate z estimate, obtained from the pressure gage that also equips the IMU.

Evidently, the greater the number of auxiliary sources of measurement, the better will be the estimation. Effort is being made aiming to find and join new sensors to the torpedo-pile IMU.

The measurement model is given by equation (18).

$$z = h(x) + V \quad (18)$$

For those systems in which inertial forces are much smaller than gravity force, the body tilt can be obtained through the accelerometers readings of gravity components. To do so, using quaternions descriptions, one may employ the array of equations (19).

$$h = \begin{bmatrix} 2g(ac - bd) \\ -2g(ab + bc) \\ -g(a^2 + b^2 - c^2 - d^2) \end{bmatrix} \quad (19)$$

These equations are obtained from the product of the rotation quaternion, that describes a generic attitude of the body at any given position, by the gravity vector  $[0 \ 0 \ g]^T$ . One may find more information about this product in [2].

And, in the linearized form it becomes:

$$H = 2g \begin{bmatrix} c & -d & a & -b & 0 & 0 & 0 \\ -b & -a & -d & -c & 0 & 0 & 0 \\ -a & b & c & -d & 0 & 0 & 0 \end{bmatrix} \quad (20)$$

Being the pressure gages the only source of information for the translational block, measurement equations become:

$$h = [0 \ 0 \ -1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \quad (21)$$

Since it is already linear,  $h = H$ .

ROV and AUV (Autonomous Underwater Vehicles), besides the IMU sensors, are usually equipped with DVL (Doppler Velocity Log), SBL (Short Base Line) and compasses that support the filter with the vehicle velocity, position and heading, respectively. However, these are large, sophisticated and expensive sensors; thus, they are out of the present research scope. This work intends to keep the hardware costs as low as possible as they should equip anchoring or analogous devices.

## MEASUREMENT NOISE

In order to evaluate and quantify the sensors noise, as done in [3], Allan's Variance (AVAR) will be used. The AVAR curve is calculated using expression (22).

$$AVAR(\tau) = \frac{1}{2(n-1)} \sum_i (y(\tau)_{i+1} - y(\tau)_i)^2 \quad (22)$$

Where  $\tau$  is the size of the cluster in which the data has been shared [4]. As presented in [5], the AVAR curve, or its square root – Allan's deviation, is related to the different sources of noises commonly found in inertial sensor signals. Figure 3 illustrates this relation.

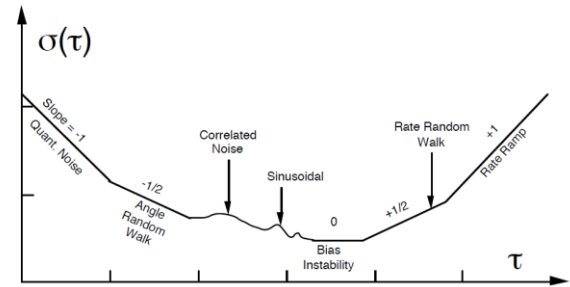


Figure 3 Allan's Variance [5]

Once one has created the AVAR – or deviation – curve and plotted it on a log-log graph, some straight line sections will arise; those sections have particular slopes and are related to the different sources of noise.

This technique not only allows one to identify the different noises on the signal, but also helps to quantify them.

EKF, as presented here, is an optimal estimator only if the noises –  $W$  and  $V$ , at equations (11) and (19) – are Gaussian. [6] presents methods to deal with other kind of noises, such as pink noises. Here, the noises will be considered white.

Sensor Random Walk is a white noise [3] and can be identified at Allan's deviation curve as a straight line with slope -1/2, see Figure 3. These straight sections were also identified in the curves crated from the signal recorded by the SMET. From its static records, the curves illustrated by Figure 4 to Figure 7 were created.

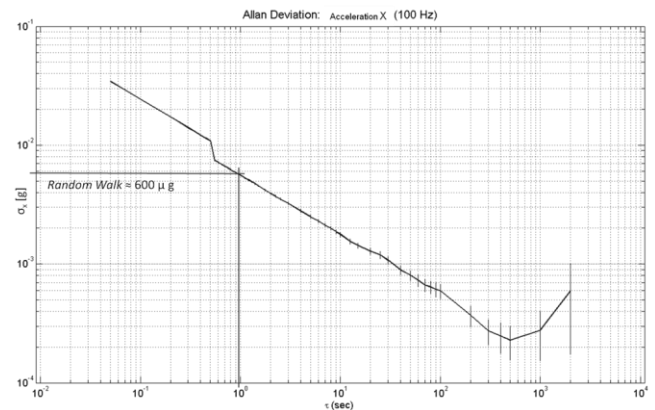


Figure 4 - Allan deviation for accelerometer x

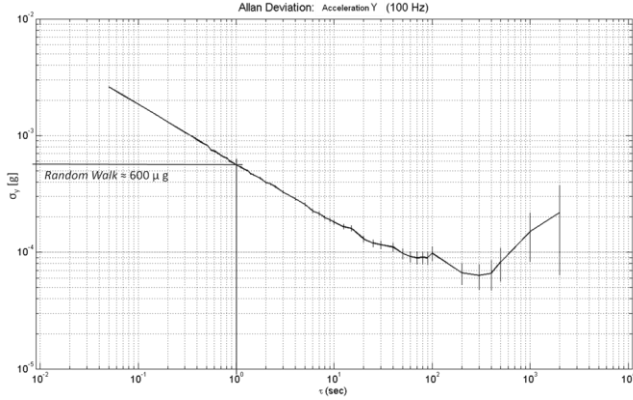


Figure 5 - Allan deviation for accelerometer y

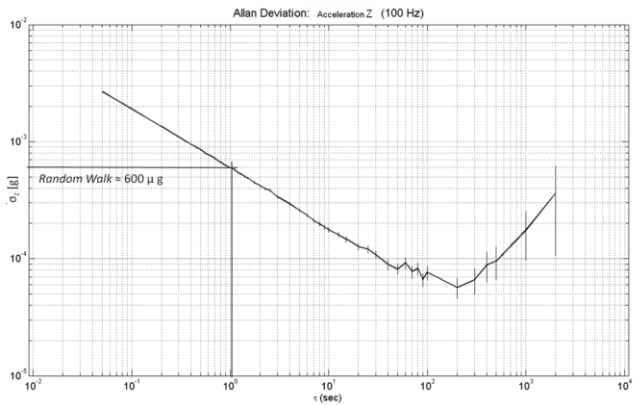


Figure 6 - Allan deviation for accelerometer z

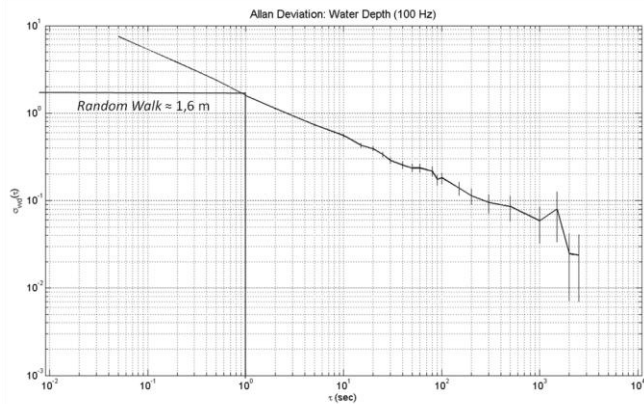


Figure 7 - Allan deviation for pressure gage (converted into water depth [m])

Table 1 summarizes the Random Walk amplitudes, extracted from the curves above.

Table 1 - Noises amplitude

Sensor	Noise Amplitude
Accelerometer x ( $\sigma_{ax}$ )	0.0006g 0.006m/s <sup>2</sup>
Accelerometer y ( $\sigma_{ay}$ )	0.0006g 0.006m/s <sup>2</sup>
Accelerometer z ( $\sigma_{az}$ )	0.0006g 0.006m/s <sup>2</sup>
Pressure Gage ( $\sigma_p$ )	1.6m

The noise covariance matrixes identified with each source of measurement are given by equations (23) and (24).

$$R_a = \begin{bmatrix} \sigma_{ax}^2 & 0 & 0 \\ 0 & \sigma_{ay}^2 & 0 \\ 0 & 0 & \sigma_{az}^2 \end{bmatrix} \quad (23)$$

$$R_t = \sigma_{wd}^2 \quad (24)$$

## PROCESS NOISE

The process noise is harder to be estimated, and can be used as a tuning parameter of the Kalman filtering, controlling the trade-off between the measurements and the dynamic model predictions.

## ALGORITHM TESTS

Before actually applying the algorithm described above to reconstruct the piles trajectory, some static tests were performed. During these tests, the studied IMU was placed on a bench, as shown in Figure 8, and no motion was imposed.

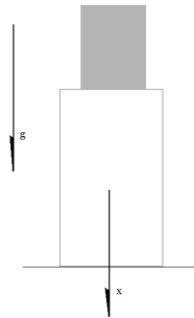


Figure 8 - Direction of sensitive axis x of the IMU

Initially, each block of the filter was tested separately. In order to observe the estimation performance, not only must the states be investigated but also the residue, represented by equation (25), and the norm of gain  $K$ .

$$(z - h(x, t)) \quad (25)$$

Although the real values of the states are known during the bench tests, in most of the situations, they are unknown and to monitor the residue is the only way to evaluate if the estimate is coherent or not. Figure 9 and Figure 10 show the estimate for the attitude states of the system.

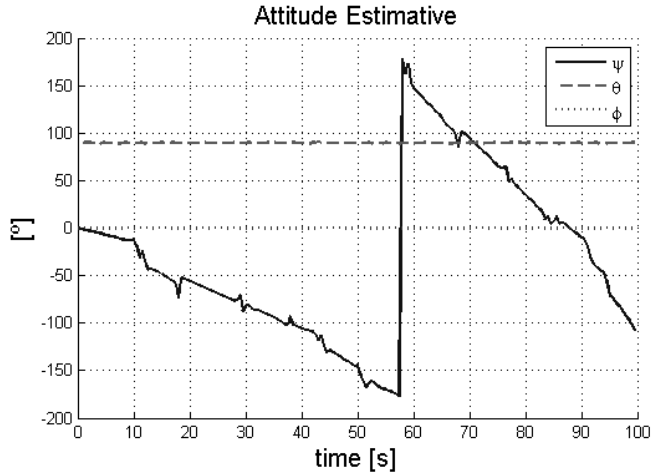


Figure 9 - Attitude estimate for the bench test signal

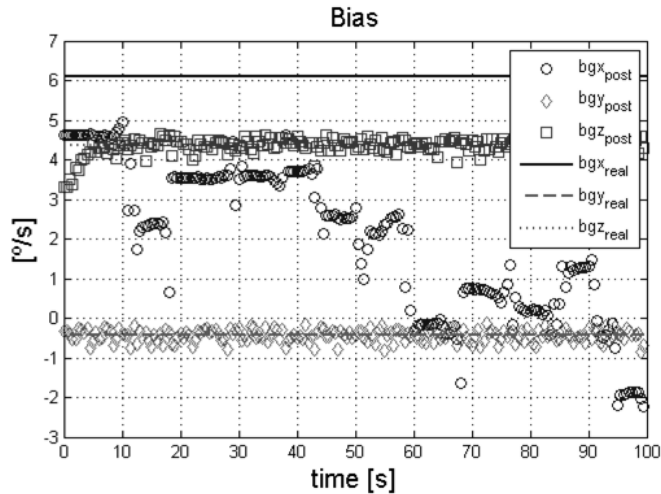


Figure 10 – Bias estimate for the bench test signal

All the estimates were expected to be constant although the lack of a heading sensor makes it impossible to estimate states  $\psi$  and the bias of rate gyro x. Continuing the analysis of results, Figure 11 brings the residue of the acceleration measurement, see equation (25).

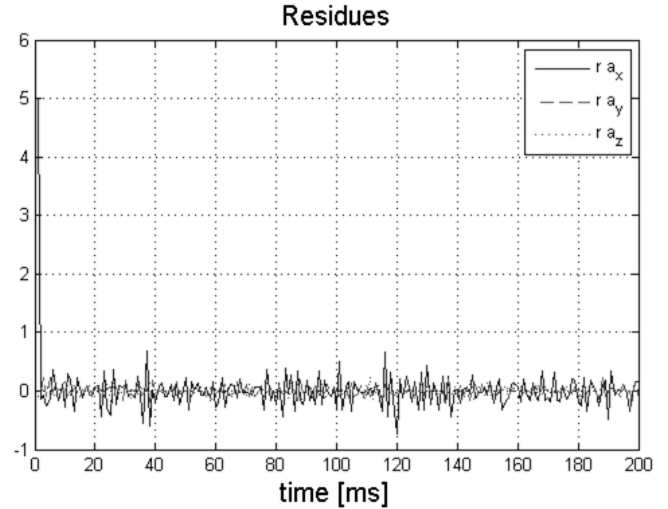


Figure 11 - Residues of each acceleration component reading

Repeating the process for the translational states estimation, leads to Figure 12. Again, all positions were expected to be constant, but the only the position z was estimated as a constant. This was the only state that had an auxiliary source of information – pressure gage.

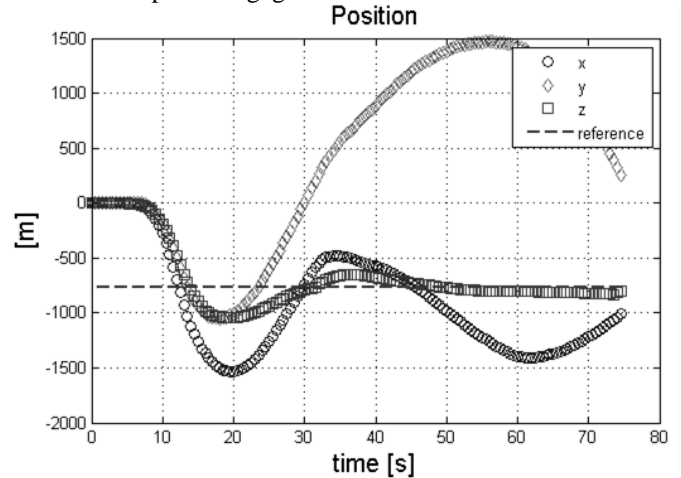


Figure 12 - Position estimate for bench test signal

This analysis showed that the auxiliary sources of measurements can, in fact, improve the quality of estimations by eliminating the drift commonly verified in inertial sensors.

### TORPEDO PILE LAUNCH SIGNAL

During the bench tests, it was observed that several parameters could not be estimated correctly as they do not have an alternative source of information. These states will be discussed later. Firstly, some interesting results will be shown.

The results below were obtained applying a real torpedo pile launch to the EKF described before.

Starting from a null condition, Attitude EKF was able to find some reasonable estimate for the gyro y and z bias, as can be seen in Figure 13.

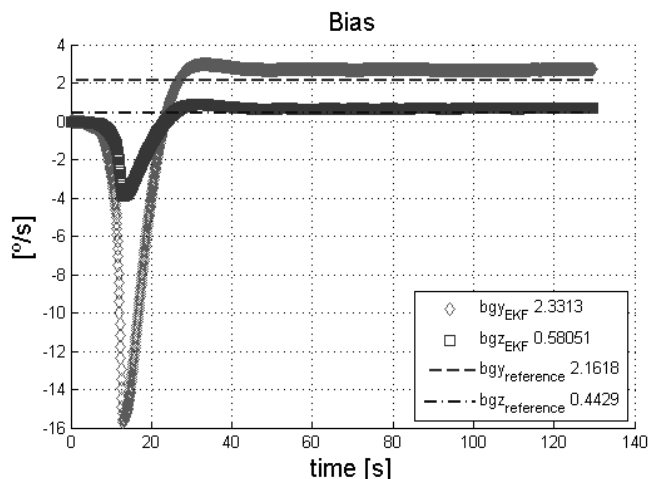


Figure 13 - Gyros bias estimate from real torpedo pile launch record

The reference values are extracted from [1]. As said previously, the reading of gravity components can only be used to calculate the unit tilt when the inertial forces are much smaller than the gravity force. From Figure 13, one can notice that during the releasing and the impact, the bias estimates are poor. On the other hand, after the torpedo stops completely, the algorithm produces good results.

Performing the integration of the trajectory using the bias estimated by Attitude EKF, it is possible to build the graph on the right of Figure 14.

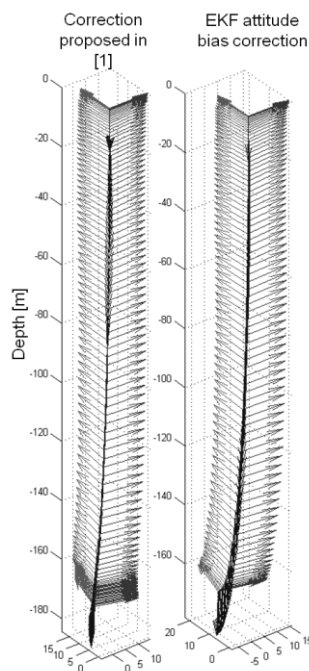


Figure 14 - Attitude of torpedo pile during the launching

## CONCLUSIONS

The EKF is a powerful tool to deal with many sources of information with distinct levels of precision and noise. Certainly, it is fundamental to reach the objective of creating a useful, rough and cheap measurement system to equip devices such as offshore anchors. However, the filter demands a careful tuning, in this case, changing the process noise covariance.

The lack of auxiliary sensors resulted in small gains as compared to the process presented in [1]. The major contribution of the EKF routine was the automation of the bias search.

## FURTHER STEPS

Work was done so as to add a magnetometer to the sensors assembly. These sensors would provide auxiliary information of the unit heading; yet, they are sensitive to surrounding metallic masses. The pressure vessel that protects the IMU during the launching is made of steel as well as the torpedo pile itself. This great steel mass would distort the magnetometer reading. Nevertheless, some companies provide sensors the calibration of which can neglect the near metallic masses. In case this calibration yields satisfactory results, the magnetometer equation will be included in array  $h$ .

## ACKNOWLEDGMENT

The authors acknowledge Petrobras for the financial support and for the motivation of this work. The authors also thank professor Celso Pupo Pesce for the technical support during the research. The first author acknowledges the Coordination for the Improvement of Higher Level -or Education- Personnel (CAPES) for the research grant.

Second author also acknowledges the National Council for Scientific and Technological Development (CNPq) for the research grant (302544/2010-0) and São Paulo Research Foundation - FAPESP (2010/15348-4).

## REFERENCES

1. LAVIERI, R. S.; TANNURI, E. A.; FUJARRA, A. C.; PESCE, C. P.; CORREA, D. C. **Low Cost Inertial Measurement Unit Application on Sub-Sea Launching - Signal Processing and Trajectory Algorithm**. Proceedings of the 29th International Conference on Ocean, Offshore and Arctic Engineering. Shanghai: ASME. 2010.
2. TITTERTON, D. H.; WESTON, J. L. **Strapdown Inertial Navigation Technology**. Reston: American Institute of Aeronautics and Astronautics, Inc, 2004.
3. EL-SHEIMY, N.; HOU, H.; NIU, X. Analysis and Modeling of Inertial Sensors Using Allan Variance. **IEEE TRANSACTIONS ON INSTRUMENTATION AND**



**MEASUREMENT**, v. VOL. 57, n. NO. 1, JANUARY 2008.

4. STOCKWELL, W. **Bias Stability Measurement: Allan Variance**. [S.l.]. 2004.
5. IEEE Standard Specification Format Guide and Test Procedure for Single Axis Interferometric Fiber Optic Gyros. IEEE Std 952. [S.l.]: [s.n.]. 1997 (R2008).
6. JAZWINSKI, A. H. **Stochastic Processes and Filtering**. New York: Academic Press, 1979.